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Finite Element Modeling Techniques: A Review

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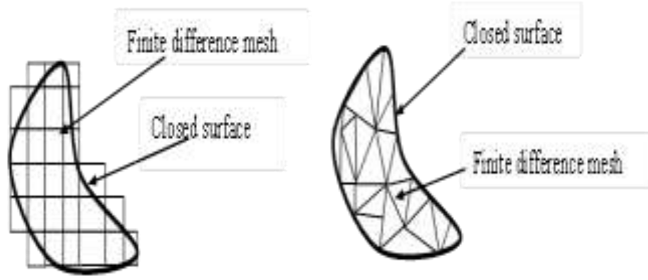
Abstract: ***Finite element modeling is used to write equation of motion a structure. A mathematical model is characterized by variables, parameters and functional forms. Euler Bernoulli beam theory, Timoshenko beam theory and higher order shear deformation theory are discussed in this paper.***

Keywords- ***Finite element modelling, discretization, beam theory.***

I. INTRODUCTION

It is quite expensive and tedious to build smart structures just to study their dynamic behaviour. The first step in active vibration control based on modern control theory is to build a mathematical model of the structure under consideration. A mathematical model is characterized by variables, parameters and functional forms. Field variables are the variables of interest governed by the differential or integral equations, parameters do not change and functional is the relation between the two. Mathematical modeling and subsequent numerical simulations greatly reduce research and development cost of smart structures. The mathematical modeling could be done by variety of analytical or numerical methods. Rayleigh Ritz method, boundary element method, principle of virtual work, finite difference method, finite element method and system identification are some of the methods used for formulating a mathematical model of the structure. In Rayleigh Ritz method some trial functions are assumed for unknown deformations or field variables. These trial functions with unknown parameters are substituted in the functional and differentiated with respect to each unknown variable. The resulting simultaneous equations are solved to get approximate solutions from the class of assumed solutions. The values of the parameters are calculated that minimize the potential energy function of the system. In boundary element method Green's theorem is used to reduce a volume problem to surface problem and a surface problem to a line problem. Finite difference method gives a point wise approximation of the governing equations. The derivative of a function at a point is approximated by an algebraic expression. Thus the system is discretized and the governing equation is replaced by an algebraic equation. The model of the system improves when more points are added. Finite difference method is suitable for uniform structures as it reduces the computations involved and thus more practical for real time applications [1]. With unusual boundary conditions or variation in cross sections as in real life structures like wings of aircrafts, communication panels etc., finite difference method becomes difficult to use. This is because more approximations are involved at the boundaries resulting in poor results (figure 1). Use of regular grids (squares and rectangles) in finite difference methods minimizes errors and makes the calculations faster. As can be seen in the Figure 1 use of regular grids at the curved boundary brings in approximation errors. As opposed to this, using simple triangular elements in FE method gives better approximation on the curved boundary. For real world structures Finite Element (FE) method is very effective in describing the dynamics of structures [2-7]. In FE method the continuous system is discretized into many small interconnected elements. In this sense FE method gives piecewise approximation against point wise approximation of finite difference method.



(a) Finite difference discretization (b) Finite element discretization

Figure 1 Finite difference discretization v/s finite element discretization

A FE mesh has a set of points called nodes. The response of a FE model is expressed in terms of values of some unknown functions at these nodes. The spatial configuration of the elements is described by the nodal displacements or generalized coordinates which are degrees of freedom of the FE model. These nodal displacements include translations and rotations at the nodes and functions need to satisfy the continuity conditions at these element nodes. The values of the field variables at the nodes are got by the approximate solution of the governing equations. Interpolation functions are used to find the value of the variables at any point in the element. Generally polynomials are used for interpolation function as they are easy to manipulate by differentiation or integration. Once the equations of motion of all the elements are formulated, they are assembled together to give the matrix equations of motion of the whole structure. Organization of paper is as: In section -II types of elements in Finite element modeling are discussed. In section -III various techniques are discussed for finite element modeling and finally in section-IV conclusions are drawn.

II. Types of Elements in Finite Element Modeling

The field of FE modeling of smart structures instrumented with piezoelectric sensors and actuators has come a long way since it was first reported in 1987 [8]. Both 2-D and 3-D elements are used for modeling of mechanical structures [9-12]. Different element types with different degrees of freedom can be chosen in the same model. Elements are classified based on their structural actions like truss element, frame element, plate element, shell element, plane element or solid element. A truss element is a two force member with one degree of freedom (axial displacement). A beam element can have upto six degrees of freedom (dofs) at each node. It is subjected to moments and lateral loads. If a beam member is considered to be under pure bending then it has only 4 dofs. A frame element is subjected to both axial and lateral loads. A plate element is a 2D solid element and has two rotational and one normal displacement as degrees of freedom at each node. A shell element is 3D solid element with very small thickness compared to other dimensions. Shell element generally has a triangular or quadrilateral shape. A plane element is used when only forces are considered but not the moments. It can be modelled as plane stress or plane strain element. General case of a solid element is an eight noded isoparametric 3D element with nine bending modes. Considering the forces and deformations involved in smart structures generally beams, plates and shell elements are used.

III. Techniques for Finite Element Modelling

Beam elements are mathematically modelled either using shear indeformable model (Euler-Bernoulli) or shear deformable model (Timoshenko). When length to thickness ratio increases beyond a particular level (15 for isotropic material and 30 for composite material) the shear effects cannot be ignored and Timoshenko beam theory is employed. In Euler Bernoulli beam theory it is assumed that the cross-sections remain plane and perpendicular to the longitudinal axis after deformation. For Euler Bernoulli beam the displacements (Fig 2) are given by:

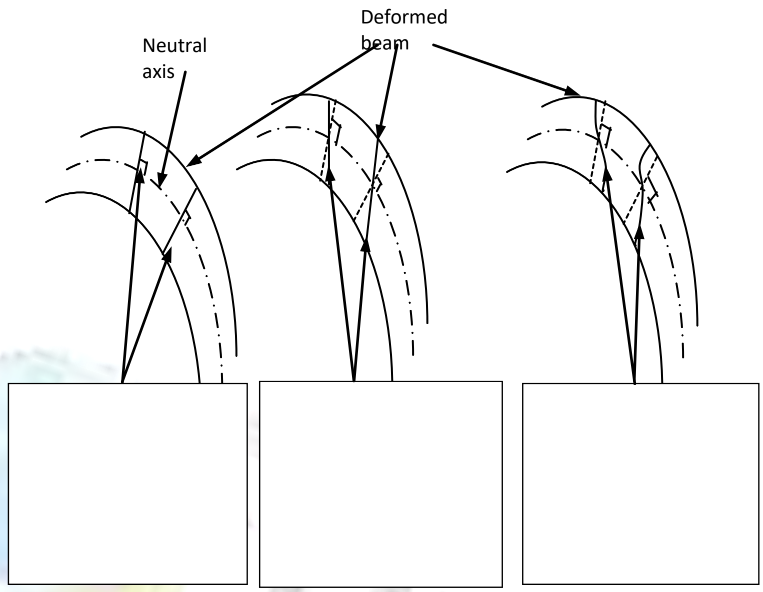
$$u(x,z,t) = -z\frac{\partial \varphi}{\partial x} \quad (1)$$

$$w(x,z,t) = \varphi(x,t) \quad (2)$$

$$v(x,z,t) = 0 \quad (3)$$

where 'u', 'v' & 'w' are the displacements in x, y & z directions, '$\varphi(x,t)$' is the field variable for pure bending and 'z' is the transverse coordinate measured from the middle plane of the beam. The longitudinal strain in the beam is given by

$$\varepsilon_x = -z\frac{\partial^2\varphi}{\partial x^2} \quad (4)$$



(a) Euler Bernoulli theory (b) Timoshenko Theory (c) Higher order theory

Figure 2 Deformation of the cross-section in different beam theories (in (b) and (c) dashed line represents the deformed section in Euler Bernoulli theory)

In Timoshenko beam theory it is assumed that the cross-section remains plane but not perpendicular to the longitudinal axis during deformation. This rotation of the cross section is assumed to be due to shear deformation which is ignored in Euler Bernoulli beam theory. If the length to thickness ratio is large i.e. thin beams then difference between the results of these two theories is negligible. In case of for short and thick beams Timoshenko beam theory gives better results. In Timoshenko beam, along with field variable for bending field variable for shear is also considered to calculate displacements (figure 2). Displacements using Timoshenko beam theory are given by

$$u(x,z,t) = z\theta(x,t) \quad (5)$$

$$v(x,z,t) = 0 \quad (6)$$

$$w(x,z,t) = w_o \quad (7)$$

where '$\theta(x,t)$' denotes the rotation of the cross section of the beam about normal to the middle axis and 'w' is the transverse displacement of the centroidal axis or neutral fibre of the beam. The net shearing force acting on the section is written as:

$$V = GA_s\theta \quad (8)$$

where '$A_s = A/k$', 'A' is cross sectional area of the beam. Here 'k' is the shear correction factor introduced to take care of the non-uniformity in the shear force across the beam section. Various shear correction factors have been presented: some are calculated by matching shear strain energy with that of first order shear deformation theory, in some strain energy from the beam theory and the equilibrium equations is equated, while some shear correction factors are calculated by comparing the results of Mindlin plate theory and exact 3D theory of elasticity etc. [13, 14]. Accuracy of the first order shear deformation theory depends on the shear correction factor. Limitation of Timoshenko beam theory is that when it is applied to very thin plates it leads to problem of excessive stiffness leading to shear locking. This effect can be taken care of by using reduced integration or mixed formulation [15]. To improve the accuracy of response higher order deformation theories are also considered assuming parabolic variation of transverse shear stresses [16, 17]. Higher order shear deformation theories do not need shear correction factor. The displacements with higher order shear correction theories are given by:

$$u(x,z,t) = u_0(x,t) + zu_1(x,t) + z^2u_2(x,t) + \cdots \quad (9)$$

$$w(x,z,t) = w_0(x,t) + zw_1(x,t) + z^2w_2(x,t) + \cdots \quad (10)$$

where coefficients '$u_i(x,t)$'in the axial displacement expression are independent unknowns to be determined and coefficients '$w_i(x,t)$' in the transverse displacement equation are the kinematic variables in the transverse direction. The degree of the polynomial is chosen appropriately to capture the higher order deformation effects

IV. CONCLUSIONS

In this paper a review of mathematical modeling using finite element method is presented. Effectiveness of finite element modeling in describing the dynamics of thea structure is compared with other mathematical modeling techniques. It is concluded that finite element method is a superior modeling method for structures. Later the different types of elements used in modeling are discussed in detail. In the next section detailed comparison of different techniques of finite element modeling of beams are presented with their relative merits and demerits. Thus this review would help in the choice finite element modeling techniques for structures.

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