

# Two phase hybrid AI-heuristics for Multiple travelling salesman problem

<sup>[1]</sup> N.Sathya, <sup>[2]</sup> Dr.A.Muthukumaravel,

<sup>[1]</sup> Research Scholar, Bharathiar University, Coimbatore, India.

<sup>[2]</sup> Professor & Head, Department of MCA, Bharath Institute of Higher Education and Research, Chennai, India.

*Abstract: A multiple Vehicle Routing Problem (mVRP), which consists of determining a set of routes for m vehicles who all start from and return back to a home city (depot). Given a set of cities, let there are m vehicles located at a single depot node. The remaining cities that are to be visited are called intermediate nodes. SP Routing concept is the primary component in the performance of any network. The Shortest path between any two nodes is required in most of the real world oriented network problems. The objective of this paper is to develop efficient optimization algorithms to solve diverse real world SP routing network problems of high complexity, to find the shortest possible balanced routes for multiple salespersons and to find clustering based simple straight-forward hierarchical SP routing in solar powered WSNs.*

Keywords :

## I. INTRODUCTION

United Parcel Service (UPS), the world's largest package delivery company, saved around \$276 million over a decade after it had restructured its overnight delivery network using the concepts of advanced network optimization (Chen 2007). The growing pressure for cost reduction due to increasing fuel and labor costs. The need for alternative efficient techniques to replace the human beings in border surveillance at difficult geographical terrains and the continuous advancements in computing technology.

## II. SP PROBLEM

Shortest Path: The shortest path between two vertices 's' and 't' in a network is a simple path from 's' to 't' with the property that no other such path has a lower weight Bondy and Murty (1982). Shortest Path Problem: Many optimization problems amount to finding a sub-graph of certain type with minimum weight in a weighted graph. One such is the Shortest Path Problem (SPP). Example: a road map network connecting various towns.

## III. PROBLEM DESCRIPTION

Among the various graph algorithms, the SP algorithm and Hamiltonian cycle algorithm (VRP) have a lot of applications in various real world situations. The following are the three important problems based on the afore-mentioned graph algorithms. Multiple Vehicle Routing problem, Distribution Network Problems and Shortest path routing in multi-hop solar powered wireless sensor networks.

## IV. PROPOSED ALGORITHMS

K-means clustering based two stage hybrid AI-heuristics are proposed for routing in mTSPs and WSNs. For mTSPs: k-means clustering based two phase hybrid AI-heuristics algorithm is proposed in this paper. The goal of this thesis is to develop efficient optimization algorithms to solve diverse real world SP routing network problems of high complexity. To find the shortest possible balanced routes for multiple salespersons. To find clustering based simple straight-forward hierarchical SP routing in solar powered WSNs.

A) k-means clustering and GA

In the first phase of the algorithm, all the cities are grouped into number of clusters by using k-means clustering algorithm. In the second phase of the algorithm, the clustered cities are provided as input to GA and ACO algorithms. By using the input provided to them, GA and ACO algorithms compute the improved routes for each cluster.

**B) Assumptions**

All the salespersons have to start from a common depot and after traveling through a set of cities, they should return back to the starting depot. All the cities are completely connected with other cities using symmetrical links. Between any two cities, only one symmetrical link exists. There are no capacity constraints and no cost constraints.

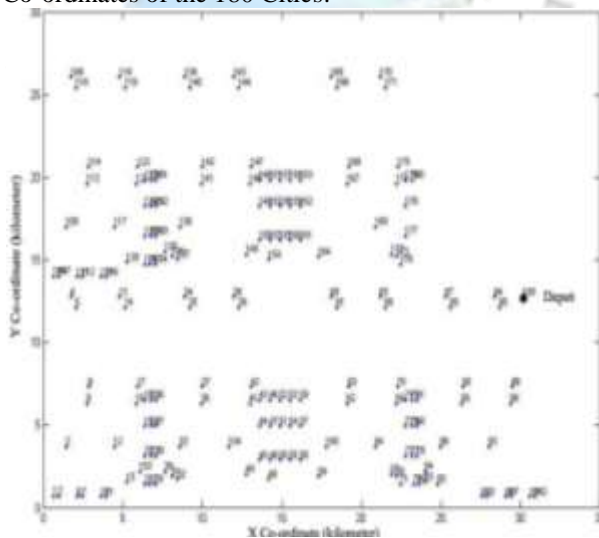
**V. TWO PHASE GA**

To demonstrate the proposed two phase AI-heuristic methods, a mTSP with 180 cities and 6 salespersons is considered. City 100 is assumed as depot.

Phase 1: k-means clustering, Phase 2 : Genetic Algorithm

| Computing term                     | Biological (GA) term   |
|------------------------------------|------------------------|
| Solution to a problem              | Individual             |
| Set of solutions                   | Population             |
| Quality of a solution              | Fitness                |
| Encoding of solution               | Chromosome             |
| Part of the encoding of a solution | Gene                   |
| Search operators                   | Crossover and mutation |
| Reuse of good solutions            | Natural selection      |

Co-ordinates of the 180 Cities:



The steps in k-means clustering:

Step 1: Begin with the value of number of clusters k. (k = m = 6).

Step 2: Choose k initial cluster centers Z1, Z2, ..., Zk randomly from the 'N' cities {x1, x2, ..., xN}.

Here, it is considered that the depot (city 100) itself as an initial centroid. the cities 98, 99, 100, 109, 110 and 118 are considered as initial cluster centers.

Step 3: The cities are arranged based on distance calculated using the Euclidean distance formula,

$$\sqrt{[(x_1 - x_2)^2 + (y_1 - y_2)^2]}$$

In the formula, suffixes 1 & 2 represent the cities between which distances are to be calculated by using x and y co-ordinates. Assign city  $x_i$ ,  $i = 1, 2, \dots, N$  to cluster  $C_j$ ,  $j = \{1, 2, \dots, k\}$  if and only if  $\|x_i - Z_j\| < \|x_i - Z_p\|$ ,  $p = 1, 2, \dots, k$ , and  $j \neq p$ . Ties are resolved arbitrarily.

Considering cities 99 (29.45, 7.32) and 100 (30.16, 12.76) as centroids  $C_1$  and  $C_2$  and city 25 (6.77, 6.54).

$$= 22.6934 \text{ km and}$$

$$= 24.2029 \text{ km}$$

$$\frac{\sqrt{[(6.77-30.16)^2 + (6.54-12.76)^2]}}{\sqrt{[(6.77-29.45)^2 + (6.54-7.32)^2]}}$$

Based on these values, city 25 will be allotted to cluster 1.

Step 4: After assigning all the cities into various clusters, update the centroid of the cluster gaining the new cities and the cluster losing the cities. Compute new cluster centers  $Z_1^*$ ,  $Z_2^*$ , ...,  $Z_k^*$  as follows: Where  $n_i$  is the number of cities belonging to cluster  $C_i$ . Assuming the cities 29,30,31,32 and 33 belong to a particular cluster with cluster center  $Z$  (8.0,2.0), the new cluster center can be calculated by using equation.

$$Z^* = (1/5)[(7.01+7.64+8.11+8.43+8.58), (1.42+2.2+1.8 \quad 9+1.73+3.7)] = (7.954, 2.188)$$

Therefore (7.954, 2.188) is the new cluster center.

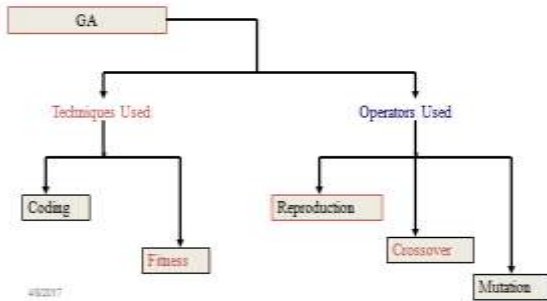
Step 5. Repeat step 3 and 4 until convergence is achieved, that is, until a pass through the training sample causes no new assignments. If  $Z_i^* = Z_i$ ,  $i = 1, 2, \dots, k$  then terminate. Otherwise continue from step 3. From the above example,  $Z^* \neq Z$ . Therefore, the algorithm repeats from step 3 by considering  $Z^*$  as the new cluster center.

*A) Results of k-means Clustering*

| Cluster/<br>Salesman | Cities allocated  |
|----------------------|---|
| 1                    | 138 35 109 124 135 5 112 118 140<br>137 127 108 123 107 132 133 136<br>4 116 119 120 121 110 117 34 129<br>122 111 106 131 115 142 126 113<br>141 114 14 13 125 130 128 38 139<br>134 |
| 2                    | 144 145 169 66 159 173 155 167<br>163 177 154 60 176 65 170 172<br>171 175 148 165 161 143 152 158<br>160 153 146 149 147 174 151 157<br>168 150 178 180 164 156 61 166<br>162 179    |
| 3                    | 104 39 63 62 50 105 42 46 55 48<br>40 49 47 54 51 45 53 41 43 57 59<br>52 56 58 44  |
| 4                    | 19 24 33 103 32 27 22 20 15 30<br>21 16 8 25 18 26 17 12 23 36 29<br>31 28 9 37   |
| 5                    | 2 7 10 11 3 6 1   |
| 6                    | 97 92 84 69 99 77 80 74 86 82 68<br>90 81 72 73 75 71 64 70 87 88 78<br>96 93 89 79 85 102 95 76 83 91<br>101 67 98 94  |

**VI. GA FOR mTSP:**

In the second phase of the algorithm, GA is applied to each cluster by treating it as a simple TSP.



Encoding:

|     |    |   |   |   |    |   |   |     |
|-----|----|---|---|---|----|---|---|-----|
| 100 | 11 | 3 | 6 | 1 | 10 | 7 | 2 | 100 |
|-----|----|---|---|---|----|---|---|-----|

Chromosome-A

|     |    |    |   |   |   |   |   |     |
|-----|----|----|---|---|---|---|---|-----|
| 100 | 10 | 11 | 3 | 2 | 7 | 6 | 1 | 100 |
|-----|----|----|---|---|---|---|---|-----|

Chromosome-B

Fitness value: Fitness value or function  $f_i = 1/d_i$ , Where  $d_i$  is the distance travelled by a salesman after covering all the cities allocated to him and  $i$  represents the chromosome number.

Selection or Reproduction: Based on the fitness values 7 sequences are selected from 10 chromosomes.

**GA - Partially Matched Crossover:**

|     |    |   |   |   |    |   |   |     |
|-----|----|---|---|---|----|---|---|-----|
| 100 | 11 | 3 | 6 | 1 | 10 | 7 | 2 | 100 |
|-----|----|---|---|---|----|---|---|-----|

Parent Chromosome-A

|     |    |    |   |   |   |   |   |     |
|-----|----|----|---|---|---|---|---|-----|
| 100 | 10 | 11 | 3 | 2 | 7 | 6 | 1 | 100 |
|-----|----|----|---|---|---|---|---|-----|

Parent Chromosome-B

a) Before PMX with two crossover indices

|     |    |   |   |   |   |   |   |     |
|-----|----|---|---|---|---|---|---|-----|
| 100 | 11 | 3 | 3 | 2 | 7 | 7 | 2 | 100 |
|-----|----|---|---|---|---|---|---|-----|

Chromosome-A1

|     |    |    |   |   |    |   |   |     |
|-----|----|----|---|---|----|---|---|-----|
| 100 | 10 | 11 | 6 | 1 | 10 | 6 | 1 | 100 |
|-----|----|----|---|---|----|---|---|-----|

Chromosome-B1

b) After exchange of crossover regions

|     |    |    |   |   |   |    |    |     |
|-----|----|----|---|---|---|----|----|-----|
| 100 | 11 | H1 | 3 | 2 | 7 | H2 | H3 | 100 |
|-----|----|----|---|---|---|----|----|-----|

Chromosome-A1a

|     |    |    |   |   |    |    |    |     |
|-----|----|----|---|---|----|----|----|-----|
| 100 | H4 | 11 | 6 | 1 | 10 | H5 | H6 | 100 |
|-----|----|----|---|---|----|----|----|-----|

Chromosome-B1a

c) After exchange of crossover regions with holes

|     |    |   |   |   |   |    |   |     |
|-----|----|---|---|---|---|----|---|-----|
| 100 | 11 | 6 | 3 | 2 | 7 | 10 | 1 | 100 |
|-----|----|---|---|---|---|----|---|-----|

Child Chromosome-A2

|     |   |    |   |   |    |   |   |     |
|-----|---|----|---|---|----|---|---|-----|
| 100 | 7 | 11 | 6 | 1 | 10 | 3 | 2 | 100 |
|-----|---|----|---|---|----|---|---|-----|

Child Chromosome-B2

d) After PMX

By considering the parent of the alternate chromosome, the holes in A1 and B1 can be filled. Considering H1 in A1 at gene position 3, the missing city number at this position is 3. Check the gene position that contains a city number 3 in B. The value 3 appears at gene position 4 in B. Fill H1 in A1 with a city number found at gene position 4 in A. The same procedure is used to fill the other holes in A1 and B1.

GA – Mutation (Inversion)

|     |    |   |   |   |   |    |   |     |
|-----|----|---|---|---|---|----|---|-----|
| 100 | 11 | 6 | 3 | 2 | 7 | 10 | 1 | 100 |
|-----|----|---|---|---|---|----|---|-----|

Chromosome-A2

|     |    |    |   |   |   |   |   |     |
|-----|----|----|---|---|---|---|---|-----|
| 100 | 11 | 10 | 3 | 2 | 7 | 6 | 1 | 100 |
|-----|----|----|---|---|---|---|---|-----|

Chromosome-A3

After performing the inversion on the selected chromosomes, the next set of 10 chromosomes is taken as the initial population for the next iteration. The set consists of: Chromosomes which are genetically modified by crossover, Chromosomes which are genetically modified by inversion, Chromosomes not selected for crossover. The steps are repeated for several iterations till the fitness value of the given population converges to a constant value. The same procedure of GA is repeated for remaining clusters to get the optimized sequences for other salespersons.

### VII. RESULTS OBTAINED AFTER PERFORMING GA :

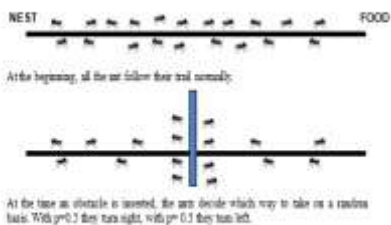
#### Results Obtained after Performing GA

| Sales Person | Route generated  | Distance to travel (km) |
|--------------|--|-------------------------|
| 1            | 100-119-109-110-118-130-123-121-139-140-128-125-111-116-115-4<br>-106-107-112-5-134-137-135-35-34-38-138-14-13-120-127-126-117<br>-113-108-124-129-133-114-132-122-141-131-136-142-100 | 182.102                 |
| 2            | 100-159-178-180-175-174-179-170-144-143-148-152-161-177-150-<br>145-153-149-160-154-155-156-151-163-162-169-167-168-171-166-<br>165-146-147-158-157-173-172-176-66-65-61-60-164-100    | 154.978                 |
| 3            | 100-62-59-105-57-54-50-49-56-55-48-45-40-104-39-41-43-52-58-<br>53-46-42-44-47-51-63-100   | 76.368                  |
| 4            | 100-22-21-12-103-15-29-31-32-33-27-18-9-8-16-25-26-17-37-36-<br>19-24-20-30-28-23-100  | 93.048                  |
| 5            | 100-11-3-1-2-6-7-10-100  | 68.348                  |
| 6            | 100-94-69-89-85-91-101-97-96-102-92-84-79-71-77-73-75-80-64-<br>70-74-82-68-67-76-93-95-89-89-80-81-72-86-78-83-87-88-100  | 115.618                 |

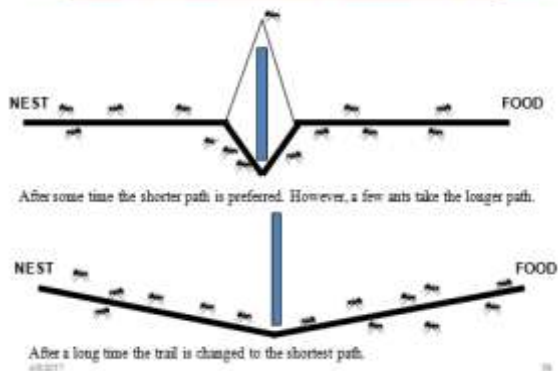
### VIII. ANT COLONY OPTIMIZATION FOR mTSP

Ant Colony Optimization (ACO) studies artificial systems that take inspiration from the behavior of real ant colonies. Stigmergy: Two individuals interact indirectly when one of them modifies the environment and the other responds to the new environment at a later time.

#### Naturally Observed Ant Behavior



### Naturally Observed Ant Behavior Contd..



The cities of each cluster can be considered as a complete graph  $G = (V, E)$ .  $d_{ij}$  is the distance associated with an edge  $(i, j)$  in the graph  $G$ . Two important parameters used in ACO to select the next city to be visited by an ant, pheromone trail  $\tau_{ij}$  and heuristic information value  $\eta_{ij} = 1/d_{ij}$ . Route construction and pheromone update are the two main steps involved in standard ACO algorithms.

#### A) ACO for mTSP -Route Construction

General steps in ACO:

- 1) Select a start city at which the ant is positioned according to some criterion.
- 2) Utilize heuristic information and pheromone values to probabilistically construct a tour (route) by iteratively adding cities that the ant has not visited so far until all the cities have been visited.
- 3) Go back to start city.

The pheromone values are initiated on all the edges of the network (Dorigo and Stutzle, 2004).

$$\tau_0 = n/R_{nn}$$

From the current city  $i$  an ant 's' can select a city  $j$  to visit among the unvisited cities based on pseudorandom proportional rule Bell and McMullen (2004):

$$j = \begin{cases} \arg \max \{ [\tau_{iu}] [\eta_{ij}]^\beta \}, & u \notin M^s, \text{ if } q \leq q_0 \\ Y & \text{if } q > q_0 \end{cases}$$

Where,  $\beta$  – a parameter,

$M^s$  – the cities already visited by the ant  $s$ ,

$q$  – a random uniform variable  $[0, 1]$ ,

$q_0$  – a parameter  $[0, 1]$ ,

$Y$  – is a random variable based on the probability equation. It gives the most probable city to be selected from a set of unvisited cities by the ant  $s$  from its current location  $i$ . If  $q > q_0$  then the next city  $j$  will be selected based on the probability value  $p_{ij}$ . The random path selection is used to select the next city. Based on the rule, the probability with which the ant  $s$  currently at city  $i$  chooses to go to city  $j$  is,

$$p_{ij}^s = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{u \in N_i^s} [\tau_{iu}]^\alpha [\eta_{iu}]^\beta}, \text{ if } j \in N_i^s$$

Where  $\alpha$  and  $\beta$  are two parameters which determine the relative influence of the pheromone trail and the heuristic information values. By using the equations, the ant will select the next city based on favorable path or randomly. Example: considering cluster 5,  $n = 8$  and assuming  $R_{nn} = 68$ , the value of  $\tau_0$  for all the edges is:  $\tau_0 = n/R_{nn} = 8/68 = 0.18$ . Assume depot (city 100) and 5 other cities 2, 3, 6, 10 and 11 in cluster 5.  $q_0 = 0.9$ . The random numbers for the cities are assumed as 0.25, 0.91, 0.98, 0.92 and 0.56 respectively. By using the condition  $q \leq q_0$ , the next city will be selected from cities 2 and 11. The Euclidean distances between depot and cities are,  $d(\text{depot}, 2) = 31.61$  km and  $d(\text{depot}, 11) = 28.87$  km. Based on pseudorandom proportional rule equation, the next city will be selected. Based on the literature:  $\alpha = 1$ ;  $\alpha = 2$ ; and  $\alpha = 0.1$ . Heuristic (depot, 2) =  $[\tau_{ij}][\eta_{ij}]^\beta = [0.18][1/31.61]^2 = 0.00018$   
 Heuristic (depot, 11) =  $[0.18][1/28.87]^2 = 0.00022$

Heuristic (depot, 11) > Heuristic (depot,2). Therefore, city 11 will be selected as next city. Considering city 2 as current city of ant s, after its visits to cities 11 and 2, the next city will be selected as follows:q value for cities 3, 6 and 10 > q0. Therefore, any one of the city will be selected based on the probabilistic rule.Distance (2, 3) = 3.03 km; Distance (2, 6) = 1.19 km; Distance (2,10) = 2.68 km  

$$p_{2,3} = \frac{([0.18]1x[1/3.03]2)}{([0.18]1x[1/3.03]2) + ([0.18]1x[1/1.19]2) + ([0.18]1x[1/2.68]2)} = \frac{0.0196}{(0.0196+0.1271+0.0251)} = 0.0196/0.1718 = 0.1141$$

Similarly,  $p_{2,6} = 0.7398$ ;  $p_{2,10} = 0.1461$ . From the above values, city 3 has 11.41% chance; city 6 has 73.98% chance and city 10 has 14.61% chance to select as next city by ant s. The probability range 0-0.1141, 0.1142-0.8539 and 0.8540-1.0 are used to select a next city. A random number (0,1) will be generated and based on the random number value the next city will be selected. For example, if the random number value is 0.43, then city 6 will be selected as next city.

**B )ACO Updating Pheromone Trails:**

Local Pheromone Update: During the route construction by an ant s, the pheromone trails on the edges it has visited are updated. The pheromone value on all edges of route Rs will be lowered by a constant factor.

Where,  $\rho_l$  is the local pheromone evaporation rate. The local pheromone update concept reduces the chances of the repeated selection of the edges by other ants. In the same manner all the ants will construct their routes.

Global Pheromone Update: After completion of the routes construction by all ants x, pheromone will be added on the edges of best path generated by one of x ants. The quantity of pheromone trail to be added on the edges can be calculated as follows:

Where  $\Delta \tau_{ij}$  is the quantity of pheromone an ant leaves on the edge it has visited and  $\rho_g$  is global evaporation rate.

$$\tau_{ij} \leftarrow \begin{cases} (1 - \rho_g)\tau_{ij} + \rho_g \Delta \tau_{ij} & \text{if edge } (i, j) \text{ belongs to } R_{best} \\ 0, & \text{otherwise} \end{cases}$$

Where  $R_{best}$  is the length of the best route generated so far by one of x ants. From the above equation, it indicates that the better an ant's route is, the ants belonging to this route will receive the more pheromone. In general, the edges that are used by more number of ants and which are lie on short routes get high quantity of pheromone. Due to more pheromone deposits, these edges are more likely to be selected by ants in the future iterations of the ACO. These steps complete one iteration. The same procedure will be repeated for preset number of iterations. Based on the literature, the values for various parameters of ACO are used as follows:  $\alpha = 1$ ;  $\beta = 2$ ;  $\rho = 0.1$ ;  $x = \text{number of cities in a cluster}$  and  $q_0 = 0.9$ .

**C) Results Obtained after Performing ACO:**

**Results Obtained after Performing ACO**

| Sales Person | Route generated  | Distance to travel (km) |
|--------------|--|-------------------------|
| 1            | 100-38-35-34-137-136-135-134-133-132-120-116-115-13-14-5-4-106-107-111-112-108-117-125-128-133-138-132-129-124-121-123-130-131-122-114-113-110-109-118-119-139-140-142-141-100 | 102.999                 |
| 2            | 100-65-66-61-60-164-161-160-155-153-150-145-154-149-152-156-159-162-163-158-157-151-148-146-147-144-143-165-166-170-171-168-167-169-175-174-179-180-178-177-172-173-176-100    | 79.3847                 |
| 3            | 100-39-42-41-43-46-52-53-58-57-54-51-47-44-104-40-49-45-48-50-55-56-59-105-62-63-100   | 60.2804                 |
| 4            | 100-33-32-31-30-29-22-21-15-103-20-23-28-27-24-19-12-8-9-17-16-18-25-26-37-36-100  | 71.3816                 |
| 5            | 100-3-1-2-6-7-10-11-100  | 65.4017                 |
| 6            | 100-99-98-93-102-101-97-96-92-91-85-83-82-78-71-68-67-64-72-77-79-84-86-80-76-73-69-74-75-81-70-89-90-88-87-94-95-100  | 53.478                  |

**D) Comparison of Results Obtained from ACO and GA**

| Salesman        | Distance obtained (km)         |                     |                      |
|-----------------|--------------------------------|---------------------|----------------------|
|                 | ACO                            | GA (300 iterations) | GA (2500 iterations) |
| 1               | 102.999                        | 182.102             | 118.7448             |
| 2               | 79.3847                        | 154.978             | 100.8326             |
| 3               | 60.2804                        | 76.368              | 64.9029              |
| 4               | 71.3816                        | 93.048              | 73.3939              |
| 5               | 65.4017                        | 68.348              | 65.4017              |
| 6               | 53.478                         | 115.618             | 62.9265              |
| Total distance  | 432.9254                       | 690.462             | 486.2024             |
| Time complexity | Algorithm execution time (Sec) |                     |                      |
|                 | 1593                           | 97                  | 532                  |

## IX. CONCLUSION

Firstly, the mTSP has been investigated. The k-means clustering based ACO performs well for large sized problems also. Computational results indicate that the proposed hybrid GA performs better than the other algorithms. The results show that AI-heuristics based route balancing concept is useful in balancing the routes. The small sized clusters reduce the message and computational overheads at the nodes as well as at CHs. The route computational time of the proposed GA is much shorter than that of Dijkstra's algorithm. The simulation results show that the algorithms are insensitive to the variations in the network topologies. From the results of various proposed algorithms, it can be concluded that the hybrid AI-heuristics algorithms are efficient concepts for various complex real world oriented shortest path routing problems.

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