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SLIDING MODE THEORY AND APPLICATION

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Abstract: ***This paper focuses mainly on the classical Variable Structure Control (VSC),also known as Sliding Mode Control (SMC) theory. Firstly, the paper introduces the classical principle of sliding mode control method. Standard sliding modes provide for finite-time convergence, precise keeping of the constraint and robustness with respect to internal and external disturbances. The practical implementation of the proposed approach is exemplified with two simple examples. Step by step design approach is developed and explained. In the first example, a linear sliding function is considered for design of sliding mode controller and in the second example a non-linear sliding function is considered to high light their effectiveness of finite-time convergence and robustness with respect to internal and external disturbances.***

Keywords: ***Variable Structure Control, Sliding Mode Control, robustness, mismatch, trajectory, switching function, sliding manifold..***

I. INTRODUCTION

In the formulation of any control problem there will, typically, be discrepancies between the actual plant and the mathematical model developed for controller design. This mismatch may be due to the variation in system parameters or the approximation of complex plant behavior by a straightforward model. It must be carefully ensured that the resulting controller has the ability to produce the required performance levels in practice despite of the plant/model mismatches. This aroused intense interest in the development of robust control methods, which seeks to solve this problem.
The sliding mode (SM) control, known also as the variable structure control, is nonlinear method of the feedback control. SM control is realized by switching of the feedback discontinuous in time between at least two
smooth functions. Therefore, the structure of the control law changes due to the location of the state trajectory in the state space. The most common SM control method is the relay in the feedback. This relay switches according some switching function. The structure of the switching function is designed in order to attract the trajectory in the state space to the switching surface. This is the manifold, where switching function equals zero. The part of the state space, where the state trajectory slides along this switching surface is called sliding mode,and represents the controlled system behavior. The advantages of obtaining such a motion are twofold, firstly the system behaves as a system of reduced order with respect to the original plant and the basic SM control algorithm provides finite time convergence of the switching function to the switching surface, and secondly the movement on the sliding surface of the system is insensitive to a particular kind of perturbation and model uncertainties. This latter property of invariance towards so-called matched uncertaintiesis the most distinguish feature of sliding mode control and makes this methodology particular suitable to deal with uncertain nonlinear systems.
Theoretical development and application to uncertain mechanical Systems, Variable structure control (VSC) with sliding mode control was first proposed and elaborated by several researchers from the former Russia, startingfrom the sixties (Emel'yanov and Taran, 1962; Emel'yanov, 1970; Utkin,The ideas did not appear outside of Russia until the seventies when a book by Itkis (Itkis, 1976) and a survey paper by Utkin (Utkin, 1977)were published in English. Since then, sliding mode control has developedinto a general design control method applicable to a wide range of systemtypes

including nonlinear systems, MIMO systems, and discrete time models, large-scale and infinite-dimensional systems.

II. Classical Theory of Sliding Mode Control

2.1.1Problem statement

Consider the following nonlinear system affine in the control
$\dot{X}$ (t) = f(t, x) + g (t, x)u(t) (2.1)
where x(t) ∈ $\Re^{n}$, u(t) ∈ $\Re^{m}$, f(t; x) ∈ $\Re^{nxn}$,
and g(t; x) ∈ $\Re^{nxm}$.
The component of the discontinuous Feedback are given by

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$u_i = \begin{cases} u_i^+(t,x) if \sigma_i(x) > 0 \\ u_i^-(t,x) if \sigma_i(x) < 0 \end{cases}$ i = 1,2 ...m (2.2)

where σ_i(x) = 0 is the i-th sliding surface, and

σ(x) = [σ_1(x),σ_2(x) ...σ_m(x)]^T=0 (2.3)

is the (n -m)dimensional sliding manifold

The control problem consists in developing continuous functions, u_i^+andu_i^-,and the sliding surface σ(x) = 0 so that the closed-loop system (2.1)-(2.2) exhibit a sliding mode on the (n-m) -dimensional sliding manifold σ(x) = 0.

The design of the sliding mode control law can be divided in two phases:

1. Phase1: consists in the construction of a suitable sliding surface so that the dynamic of the system confined to the sliding manifold produces a desired behavior.
2. Phase 2: entails the design of a discontinuous control law which forces the system trajectory to the sliding surface and maintains it there.

The sliding surface σ(x) = 0 is a (n−m)-dimensional manifold in $\Re^n$determined by the intersection of the [m. (n-1)] sliding manifold σ_i(x) = 0.The switching surface is designed such that the system response restricted to σ(x) = 0 has a desired behaviour.

Although general nonlinear switching surfaces (2.3) are possible, linear ones are more prevalent in design (Utkin, 1977; DeCarloet al., 1988; Sira-Ramirez, 1992; Edwards and Spurgeon, 1998), Thus for the sake of simplicity, this chapter will focus on linear switching surfaces of the form

σ(x) = Sx (t) = 0 where S $\epsilon\Re^{mxn}$ (2.4)

After switching surface design, the next important aspect of sliding mode control is
guaranteeing the existence of a sliding mode. A sliding mode exists, if in the vicinity of the switching surface,σ(x) = 0, the velocity vectors of the state trajectory are always directed toward the switching surface. Consequently, if the state trajectory intersects the sliding surface, the value of the state trajectory remains within a neighborhood of {x│σ(x) = 0}. If a sliding mode exists on σ(x) = 0, then σ(x) is termed a sliding surface. As seen in Fig. 2.1, a sliding mode on σ(x) = 0 can arise even in the case when sliding mode does not exist on each of the surface σ_i(x) = 0 taken separately.

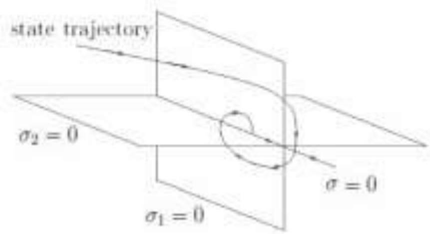

Figure2.1: Sliding mode in the intersection of the discontinuity surfaces (Resource: Slotine and Li 1991)

An ideal sliding mode exists only when the state trajectory x (t) of the controlled plant satisfies σ[x (t)] = 0 at every t ≥ t_0 for some t_0. Starting from time instant t_0, the system state is constrained on the discontinuity surface, which is an invariant set after the sliding mode has been established.

This requires infinitely fast switching. In real systems are present imperfections such as delay, hysteresis, etc., which force switching to occur at a finite frequency. The system state then oscillates within a neighborhood of the switching surface. This oscillation is called chattering. If the frequency of the switching is very high compared with the dynamic response of the system, the imperfections and the finite switching frequencies are often but not always negligible.

2.2. Existence of a sliding mode

Existence of a sliding mode (Itkis, 1976; Utkin,1977, 1992; Edwards and Spurgeon, 1998) requires stability of the state trajectory to the sliding surface σ(x) = 0 at least in a neighborhood of {x│σ(x) = 0}, i.e., the systemstate must approach the surface at least asymptotically. The largest suchneighborhood is called the region of attraction. From a geometrical pointof view, the tangent vector or time derivative of the state vector must pointtoward the sliding surface in the region of attraction (Itkis, 1976; Utkin, 1992). For a rigorous mathematical discussion of the existence of sliding modes see Itkis (1976); White and Silson (1984); Filippov(1988); Utkin (1992).

The existence problem can be seen as a generalized stability problem; hence the second method of Lyapunov provides a natural setting for analysis. Specifically, stability to the switching surface requires to choose a generalized Lyapunov function V (t; x) which is positive definite and has a negative time derivative in the region of attraction. Formally stated:

Definition 2.1 A domain D in the manifold σ = 0 is a sliding mode domain if for each ε> 0, there isδ> 0, such that any motion starting withina n-dimensional δ-vicinity of D may leave the n-dimensional δ-vicinity ofD only through the n-dimensional δ-vicinity of the boundary of D (see Fig.2.3).

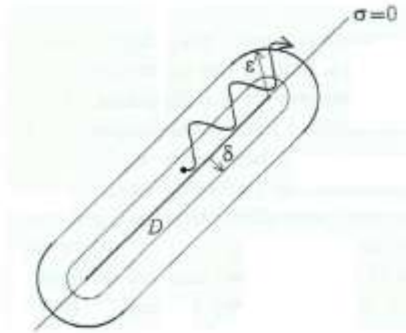

Figure 2.3: Two dimensional illustration of a sliding mode domain(Resource: Slotine and Li 1991).

Since the region D lies on the surface σ(x) = 0, dimension [D] = n- m. Hence:

Theorem 2.1 For the (n-m)-dimensional domain D to be the domain of a sliding mode, it is sufficient that in some n -dimensional domain $\Omega \in$ D, there exists a function V (t, x,σ) continuously differentiable with respect to all of its arguments, satisfying the following conditions:

V (t; x; σ) is positive definite with respect to σ , i.e., V (t; x; σ) > 0, with $\sigma \neq 0$ and arbitrary t, x, and V (t; x; 0) = 0; and on the sphere $\|\sigma\| = \rho$, for all x $\in \Omega$ and any t the relations:

$$\frac{inf}{\|\sigma\|=\rho} \text{V (t, x, } \sigma) = h_\rho, \quad h_\rho > 0 \qquad (2.5)$$

$$\frac{sup}{\|\sigma\|=\rho} \text{V (t, x, } \sigma) = H_\rho, \quad H_\rho > 0 \qquad (2.6)$$

hold, where h_ρ, and H_ρ, depend on ρ ($h_\rho \neq 0$ if $\rho \neq 0$).

2. The total time derivative of V (t, x, σ) for the system (2.1) has a negative supermum for all x $\in \Omega$ except for x on the switching surface where the control inputs are undefined, and hence the derivative of V (t; x; σ) does not exist. Proof: See Utkin (1977).

The domain D is the set of x for which the origin of the subspace(σ_1 = 0; σ_2 = 0; …….; σ_m = 0) is an asymptotically stable equilibrium pointfor the dynamic system. A sliding mode is globally reachable if the domainof attraction is the entire state space. Otherwise, the domain of attractionis a subset of the state space.

The structure of the function V (t, x,σ) determines the ease with whichone computes the actual feedback gains implementing a sliding mode controldesign. Unfortunately, there are no standard methods to find Lyapunov functions for arbitrary nonlinear systems.Note that, for all single input systems a suitable Lyapunov function is

$$V(x, t) = \frac{1}{2}\sigma^2 (x)$$

which clearly is globally positive definite. In sliding mode control, $\dot{\sigma}$ willdepend on the control and hence if switched feedback gains can be chosen so that

$$\dot{V}(t, x. \sigma) = \sigma \frac{\partial \sigma}{\partial t} < 0 \qquad (2.7)$$

in the domain of attraction, then the state trajectory converges to the surface and is restricted to the surface for all subsequent time. This latter condition is called the reaching or reachability condition (Utkin, 1992; Edwards and Spurgeon, 1998) and ensures that the sliding manifold is reached asymptotically.

Condition (2.7) is often replaced by the so-called η- reachability condition (Utkin, 1977, 1992; Edwards and Spurgeon, 1998)

$$\dot{V}(t, x. \sigma) = \sigma \frac{\partial \sigma}{\partial t} < -\eta \left| \sigma \right| < 0 \quad (2.8)$$

which ensures finite time convergence to σ(x) = 0, since by integration of(2.8) one has

$$|\sigma[x(t)]| - |\sigma x(0)| \leq -\eta t$$

showing that the time required to reach the surface, starting from the initialcondition σ[x(0)] is bounded by

$$t_s = \frac{|\sigma[x(0)]|}{\eta}$$

The feedback gains which would implement an associated sliding mode control design are straightforward to compute in this case (Utkin, 1977; Slotineand Li, 1991; Sira-Ramirez, 1992)

2.3. Existence and uniqueness of solution

The differential equations of the system (2.1) and (2.2) do not formally satisfy the classicaltheorems on the existence and uniqueness of the solutions, since theyhave discontinuous right-hand sides. Moreover, the right-hand sides usuallyare not defined on the discontinuity surfaces. Thus, they fail to satisfy conventional existence and uniqueness results of differential equation theory.

Nevertheless, an important aspect of sliding mode control design is theassumption that the system state behaves in a unique way when restrictedto σ(x) = 0. Therefore, the problem of existence and uniqueness of differentialequations with discontinuous right-hand sides is of fundamental importance. Various types of existence and uniqueness theorems can be found in Itkis (1976); Utkin (1977); and Filippov (1988).

One of the earliest and conceptually straightforward approaches is the method of Filippov (Filippov, 1988). This method is now briefly recalled asa background to the above referenced results and as an aid in understanding variable structure system behavior on the switching surface.

Consider the following n-th order single input system

$\dot{x}$(t) = f (t; x; u) (2.9)

with the following general control strategy

$$u = \begin{cases} u^{+}(t,x), & for \sigma(x) > 0 \\ u^{-}(t,x) for \sigma(x) < 0 \end{cases} \quad (2.10)$$

The system dynamics are not directly defined on the manifold σ(x) = 0. In Filippov (1988), it has been shown that the state trajectories of (2.9) with control (2.10) on σ(x) = 0 are the solutions of the equation

$\dot{x}$(t) = αf^{+} + (1 –α) f^{-}=f^{0}, 0≤ $\alpha \leq 1$ (2.11)

where f^{+} = f(t; x; u+), f^{-} = f(t; x; u^{-}), and f^{0} is the resulting velocityvector of the state trajectory while in sliding mode. The term α is a function of the system state and can be specified in such a way that the "average"dynamic of f^{0} is tangent to the surface σ(x) = 0. The geometric concept is illustrated in Fig. 2.4.

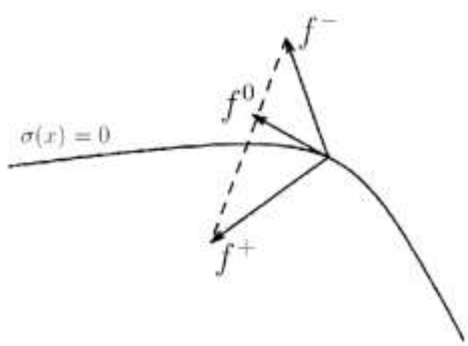

Figure 2.4: Illustration of the Filippov method (Resource: Slotine and Li 1991)

Therefore one may conclude that, on the average, the solution to (2.9) with control (2.10) exists and is uniquely defined on σ(x) = 0. This solution is called "solution in the Filippov sense". Note that this technique can beused to determine the behavior of the plant in a sliding mode.

2.4. Sliding surface design

Filippov's method is one possible technique for determining the system motionin sliding mode as outlined in the previous section. In particular, computation of f^{0} represents the "average" velocity $\dot{x}$ of the state trajectoryrestricted to the switching surface. A more straightforward techniqueeasily applicable to multi-input systems is the equivalent control method, as proposed in Utkin (1977, 1992) and in Drazenovic (1969).

It has been proved that the equivalent control method produces the same solution of the Filippov method if the controlled system is affine in the control input while the two solutions may differ in more general cases.

The method of equivalent control can be used to determine the system motion restricted to the switching surface σ(x) = 0. The analytical nature of this method makes it a powerful tool for both analysis and design purposes.

Consider the following system affine in the control input

$\dot{x}$ (t) = f (t; x) + g(t; x)u(t) (2.12)

Suppose that, at time instant t_0, the state trajectory of the plant intercepts the switching surface and a sliding mode exists for t ≥ t_0.The first step of the equivalent control approach is to find the inputu_{eq}such that the state trajectory stays on the switching surface σ(x) = 0. The existence of the sliding mode implies that σ(x) = 0, for all t ≥ t_0, and $\dot{\sigma}$ (x) = 0.

By differentiating σ(x) with respect to time along the trajectory of (2.12) it yields

$$\left(\frac{\partial\sigma}{\partial x}\right)\dot{x} = \left(\frac{\partial\sigma}{\partial x}\right)\left[f(t,x) + g(t,x)u_{eq}\right] = 0 \quad (2.13)$$

Where u_{eq} is the so-called equivalent control. Note that, under the action ofthe equivalent control u_{eq} any trajectory starting from the manifold σ(x) = 0 remains on it, since $\dot{\sigma}$ (x) = 0. As a consequence, the sliding manifold σ(x) = 0 is an invariant set.

To compute u_{eq}, let us assume that the matrix product $\left(\frac{\partial\sigma}{\partial x}\right)$g (t, x) is nonsingular for all t and x. Then

$$u_{eq} = \left[\left(\frac{\partial\sigma}{\partial x}\right) g(t;x)\right]^{-1}\left(\frac{\partial\sigma}{\partial x}\right) f(t,x) \quad (2.14)$$

Therefore, given σ[x (t_0)] = 0, the dynamics of the system on the switching surface for t≥t_0, is obtained by substituting (2.14) in (2.12), i.e.,

$$\dot{x}_1(t) = \left[I - g(t;x)\left[\left(\frac{\partial\sigma}{\partial x}\right) g(t;x)\right]^{-1}\left(\frac{\partial\sigma}{\partial x}\right)\right] f(t,x) \quad (2.15)$$

In the special case of a linear switching surface σ(x) = Sx (t), (2.15) results in

$$\dot{x}_1(t) = [I - g(t;x)[S\, g(t;x)]^{-1} S] f(t,x) \quad (2.16)$$

This structure can be advantageously exploited in switching surface design.

Note that (2.15) with the constraint σ(x) = 0 determines the system behavior on the switching surface. As a result, the motion on the switching surface results governed by a reduced order dynamics because of the set ofstate variable constraints σ(x) = 0.

2.5. Order reduction

As mentioned above, in a sliding mode, the equivalent system must satisfynot only the n-dim state dynamics (2.15), but also the m algebraicequations given by σ(x) = 0. The use of both constraints reduces the system dynamics from an n-th order model to an (n - m)-th order model.

Specifically, suppose that the nonlinear system (2.1) is in sliding modeon the sliding surface(2.3), i.e., σ(x) = Sx = 0, with the system dynamicsgiven by (2.16).

Then, it is possible to solve for m of the state variables in terms of theremaining n−m state variables, if rank[S] = m. This latter condition holdsunder the assumption that$\left(\frac{\partial\sigma}{\partial x}\right)$g (t, x) is nonsingular for all t and x.

To obtain the solution solve for m of the state variables in terms ofthe (n – m) remaining state variables. Substitute these relations into the remaining (n – m) equations of (2.16) and the equations corresponding to them state variables.

The resultant (n - m)-th order system fully describes the equivalent system given an initial condition satisfying σ(x) = 0.

III. Controller Design

The controller design procedure consists of two steps. First, a feedback control law u is selected to verify sliding condition (2.8). However, in order to account for the presence of modeling imprecision and of disturbances, the control law has to be discontinuous across σ (t). Since the implementation of the associated control switching is imperfect, this leads to chattering, chattering is undesirable in practice, since it involves high control activity and may excite high frequency dynamics neglected in the course of modeling. Thus, in a second step, the discontinuous control law u is suitably smoothed to achieve an optimal trade-off between control bandwidth and tracking precision. The first step achieves robustness for parametric uncertainty; the second step achieves robustness to high frequency unmodeled dynamics. This section discusses the first step.

Consider a simple second order system

$\ddot{x}$(t) = f (x, t) + u (t) (3.1)

where f (x, t) is generally nonlinear and/or time varying and is estimated as $\hat{f}$(x , t), u(t) is the control input, and x(t) is the state to be controlled so that it follows a desired trajectory x_d(t) . The estimation error on
f (x, t) is assumed to be bounded by some known function F=F(x, t), so that

$$\left| f(x,t) - \hat{f}(x,t) \right| \leq F(x,t) \quad (3.2)$$

we define a time varying sliding variable accordingly

$$\sigma(x) = \left(\frac{d}{dt} + \gamma\right) \tilde{x}(t) = \dot{\tilde{x}}(t) + \gamma \tilde{x}(t) \quad (3.3)$$

Differentiation of the sliding variable yields

$$\dot{\sigma}(x) = \ddot{x}(t) - \ddot{x}_d(t) + \gamma \dot{\tilde{x}}(t) \quad (3.4)$$

Substituting Equation (2.1) in Equation (3.4), we have

$$\dot{\sigma}(x) = f(x,t) + u(t) - \ddot{x}_d(t) + \gamma \dot{\tilde{x}}(t) \quad (3.5)$$

The approximation of control law $\hat{u}$(t) to achieve $\dot{\sigma}$(t)=0 is

$$\hat{u}(t) = -\hat{f}(x,t) + \ddot{x}_d(t) - \gamma \dot{\tilde{x}}(t) \quad (3.6)$$

$\hat{u}\,(t)$ can be interpreted as the best estimate of the equivalent control.
To account for the uncertainty in f while satisfying the sliding condition

$$\frac{1}{2}\frac{d}{dt}(\sigma(t)^2) \leq -\eta \left| \sigma(t) \right| \quad > 0 \quad (3.7)$$

take the control law as:

$$u(t) = \hat{u}(t) - k(x,t)\,\mathrm{sign}(s(t)) \quad (3.8)$$

By choosing k(x,t) large enough, such as

k(x, t) ?? F(x ,t) ???

ensures the satisfaction of condition (3.7), since

$$\frac{1}{2}\frac{d}{dt}(\sigma(t)^2) = \dot{\sigma}(t)\sigma(t) = (f(x,t) - \hat{f}(x,t))\,\sigma(t) - K(x,t) \left| \sigma(t) \right| \leq -\eta \left| \sigma(t) \right| \quad (3.9)$$

Hence, by using (3.8), we ensure the system trajectory will take finite time to reach the surface s (t), after which the errors will exponentially go to zero.
From this basic example, we can see the main advantages of transforming the original tracking problem into a simple 1st-order stabilization problem in s. In first-order systems, the intuitive feedback control strategy “if the error is negative, push on the positive direction; if the error is positive, push on the negative direction” works. The same statement is not true in higher-order system.
Now consider the second order system in the form of

$$\ddot{x}(t) = f(x,t) + b(x,t)\,u(t) \quad (3.10)$$

where b(x, t) is bounded as

$$0 \leq b_{min}(x,t) \leq b(x,t) \leq b_{max}(x,t)$$

The control gain b(x,t) and its bound can be time varying or state dependent. Since the control input is multiplied by the control gain in the dynamics, the geometric mean of the lower and upper bound of the gain is a reasonable estimate:

$$\hat{b}(x,t) = \sqrt{b_{min}(x,t)\,b_{max}(x,t)}$$

Bounds can then be written in the form

$$\beta^{-1} \leq \frac{\hat{b}}{b} \leq \beta \text{ where } \beta = (\beta_{max}/\beta_{min})^{1/2}$$

since the control law will be designed to be robust to the bounded multiplicative uncertainty, β is called the gain margin of the design.
It can be proved that the control law

$$u(t) = \hat{b}(x,t)^{-1}[\hat{u}(t) - k(x,t)\,\mathrm{sign}(s(t))] \quad (3.11)$$

with

$$k(x,t) \geq \beta(x,t)(F(x,t) + \eta) + (\beta(x,t) - 1)\left| \hat{u}(t) \right| \quad (3.12)$$

satisfies the sliding condition.
The control law for higher order system can be deducted based on similar approach

4.1.1 Example 1

Consider a second order nonlinear system given below

$$\dot{x}_1 = x_2$$

$\dot{x}_2 = f(x, t) + u$ (4.1)

Where f (x, t) is a nonlinear function, which can be continuous or discontinuous,

During the reaching phase, it can be observed that x_1 is stable if,

$\dot{x}_1 = -cx_1, \quad c > 0$ (4.2)

Taking sliding surface as

$S = x_2 + cx_1$ (4.3)

$\Rightarrow x_2 = -cx_1 + S$ (4.4)

Above equation is stable only when S=0

Sliding phase dynamics are elaborated by taking time derivative of S

$\dot{S} = \dot{x}_2 + c\dot{x}_1$ (4.5)

$\dot{S} = f(x, t) + u + cx_2$ (4.6)

4.1.2. Convergence analysis

The convergence is analyzed by using the Lyapunov candidate function as

$V = \frac{1}{2}S^2$ (4.7)

In order to provide the asymptotic stability of Eq. (4.7) about the equilibrium point S=0, the following conditions must be satisfied:

(a) $\dot{V} < 0$ for $S \neq 0$

(b) $\lim_{|\sigma|\to\infty} V = \infty$

Taking the time derivative of Lyapunov function following is obtained.

$\dot{V} = S\dot{S}$ (4.8)

$\dot{V} = S(f(x,) + u + cx_2)$ (4.9)

Condition (b) is obviously satisfied by V in Eq. (4.7). In order to achieve finite-time convergence (global finite-time stability), condition (a) can be modified to be

$\dot{V} \le -\alpha V^{\frac{1}{2}}, \quad \alpha > 0$ (4.10)

Indeed, separating variables and integrating inequality (4.10) over the time interval $0 \le \tau \le t$, we obtain

$V^{\frac{1}{2}}(t) \le -\frac{1}{2}\alpha t + V^{\frac{1}{2}}(0)$ (4.11)

Consequently, V(t) reaches zero in a finite time t_r that is bounded by

$t_r \le \frac{2V^{\frac{1}{2}}(0)}{\alpha}$ (4.11a)

Therefore, a control u that is computed to satisfy Eq. (4.10) will drive the variable s to zero in finite time and will keep it at zero thereafter.The derivative of V is computed as

$\dot{V} = s\dot{s} = s(f(x,\dot{t}) + u + cx_2)$ (4.12)

Assuming $u = -cx_2 + v$ and substituting it into Eq. (4.12) we obtain

$\dot{V} = s(f(x,t) + v) = s\,f(x,t) + s\,v \le |s|L + sv$ (4.12a)

Selecting $v = -\rho$ sign $(\sigma)$where

$\text{sign}(s) = \begin{cases} 1 & if\ x > 0 \\ -1 & if\ x < 0 \end{cases}$ (4.12.1)

and sign (0) $\in [-1, 1]$ (4.12.2)

with $\rho > 0$ and substituting it into Eq. (4.12a) we obtain

$\dot{V} \le |s|L + |s|\rho = -|s|(\rho - L)$ (4.12.3)

Taking into account Eq. (4.7), condition (4.10) can be rewritten as

$\dot{V} \le -\alpha V^{\frac{1}{2}} = -\frac{\alpha}{\sqrt{2}}|s|\alpha > 0$ (4.12.4)

Combining Esq. (4.12.3) and (4.12.4) we obtain

$\dot{V} \le -|s|(\rho - L) = -\frac{\alpha}{\sqrt{2}}|s|$ (4.12.5)

Finally, the control gain ρ is computed as

$\rho = L + \frac{\alpha}{\sqrt{2}}$ (4.12.6)

Consequently a control law u that drives $\sigma$to zero in finite time (4.11a) is

$u = -cx_2 - \rho\, sign\,(s)$ (4.12.7)

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Remark 4.1. It is obvious that $\dot{\sigma}$must be a function of control u in order tosuccessfully design the controller in Eq.(4.10)or (4.12.7).This observation must betaken into account while designing the variable given in Eq.(4.3).

Remark4.2. The first component of the control gain Eq. (4.12.6) is designed tocompensate for the bounded disturbance

f (x, t), while the second term $\frac{\alpha}{\sqrt{2}}$ is responsible for determining the sliding surface reaching time given by Eq. (4.11a).The larger,α, the shorter, the reaching time.

4.2.1. Example 2

Consider a simple pendulum described by the following set of equations: \

$\dot{x}_1 = x_2$

$\dot{x}_2 = b\sin x_1 + u$ (4.13)

Switching surface is taken as

$S = cx_1 + x_2$ (4.14)

and also

$\dot{S} = c\dot{x}_1 + \dot{x}_2$

$\dot{S} = cx_2 + b\sin x_1 + u$ (4.15)

For equivalent control, put $\dot{S}=0$

$cx_2 + b\sin x_1 + u_{eq} = 0$

$\Rightarrow u_{eq} = -cx_2 - b\sin x_1$ (4.16)

Then sliding mode control law is

$u = ueq - Ksign(s)$

$u = -cx_2 - b\sin x_1 - Ksign(s)$ (4.17)

4.2.2. Bound on gain K

For calculation of bound for gain K, plug in the control law (4.17) in (4.15) and get $\dot{S} = \underbrace{cx_2 + b\sin x_1 - cx_2 - b\sin x_1}_{\Delta(\text{uncertain})} -$

k sign (s)

Let the parameter b of pendulum be uncertain, therefore

$S = \Delta - k\,\text{sign}(s)$

Where Δ is the uncertainty vector, of the simple pendulum parameter, the bounds on K for negative definite of $S\dot{S}$ should be

$k > \|\Delta\|$.

Simulating the trajectories of pendulum system, let K=1, c=1 and b=0.8 and initial conditions are x_0= (0.6, 0), phase portrait of sliding motion is obtained and it is shown in Figure 4.1. The control action is depicted in Figure 4.2. Figure 4.3 demonstrates the system states, where as the sliding surface is given in Figure 4.4. Now it is clear from Figure 4.3 that system states converge in finite time. So in conclusion, it can be said that

remain. Now it is clear from Figure 4.3 that system states converge in finite time. So in conclusion, it can be said that when phase

portrait intercepts sliding surface, finite time then it is forced to remain.

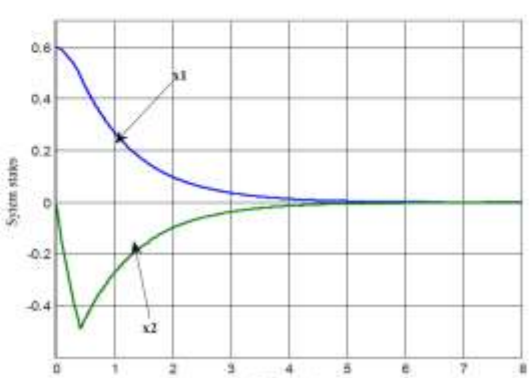

Figure 4.1: Sliding motion phase portrait

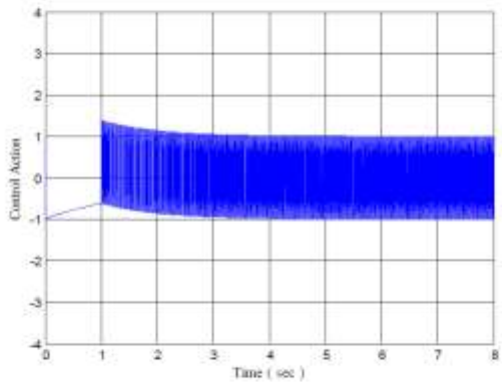

Figure4.2: Controller effort

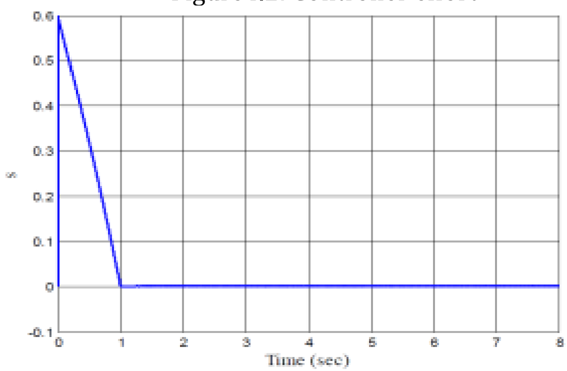

Figure4.3:SystemTrajectories

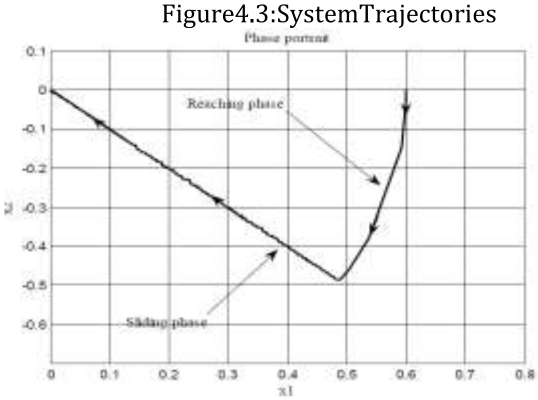

Figure4.4: Sliding Surface.

V. Conclusions:

The classical theory of Sliding Mode Control is analyzed. The presence of modeling imprecision b(x, t), can be overcome by the sliding mode controller, using non-linear sliding function σ (x) = ($\frac{d}{dt}$ +γ) $\tilde{x}$(t).

To overcome bounded external disturbance

b (x, t) , we are able to design sliding mode controller, that rejects disturbance. Both systems are found to perform robust way,

rejecting, modeling imperfections and external disturbances in a remarkable way.

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